

Finding this frequency is a matter of substituting what we know into the formula $f=c / \lambda$. We know the length of the string $(L)$, and we also know that $L=\lambda / 2$. Therefore, $\lambda=2 L$. Substituting $2 L$ for $\lambda$, the string's lowest resonant frequency is found with the formula $f=c / 2 L$. In reality, the speed of sound (c) is different for a vibrating string than it is for air. For the benefit of the math-minded student, the value of $c$ for a string equals the square root of the ratio of its tension $(T)$ to its mass $(M)$, so that the real formula for the string's resonant frequency $F_{0}$ is

$$
F_{0}=\frac{1}{2 L} \cdot \sqrt{\frac{T}{M}}
$$

Other standing waves can also develop, provided they meet the requirement that there must be a node at each end of the string. For this criterion to be met, the string must be divided into exact halves, thirds, fourths, etc., as illustrated in Fig. 1.16. These standing wave patterns are called the second, third, fourth, etc., modes of vibration. Because the segments of the second mode are exactly half of the length of first mode, they produce a frequency that is exactly twice the fundamental. If we call the fundamental the first harmonic, then the second mode produces the second harmonic. Similarly, the segments of the third mode are exactly one-third the length of the first mode, so that they produce a third harmonic that is exactly three times the fundamental. The same principles apply to the fourth mode and harmonic, and beyond.

Vibrations in tubes The column of air inside a tube can be set into vibration by various means,
such as blowing across the tube's open end. If this is done with different-size tubes, we would find that (1) shorter tubes are associated with higher pitches than longer ones and (2) the same tube produces a higher pitch when it is open at both ends compared with when it is open at one end and closed at the other.

When a column of air is vibrating in a tube that is open at both ends, the least amount of particle displacement occurs in the center of the tube, where the pressure is greatest. The greatest amount of displacement occurs at the two open ends, where the pressure is lowest. Hence, there will be a standing wave that has a displacement node in the middle of the tube and antinodes at the two ends, as illustrated in Fig. 1.17a. This standing wave pattern involves one half of a cycle in the sense that going from one end of the tube to the other end involves going from a displacement peak to a zero crossing to another peak. This trip would cover $180^{\circ}$ (half of a cycle) on a sine wave, and thus a distance corresponding to half of a wavelength. Because this longest standing wave involves half of a wavelength ( $\lambda / 2$ ), the tube's lowest resonant (fundamental) frequency must have a wavelength that is twice the length of the tube (where $\lambda=2 L$ ). For this reason, tubes open at both ends are half-wavelength resonators. In other words, the lowest resonant frequency of a tube open at both ends is determined by the familiar formula $f=c / 2 L$. We could also say that the longest standing wave pattern is the first mode of vibration for the tube and that it is related to its fundamental frequency (lowest harmonic). As for the vibrating string, each successive higher mode


Fig. 1.17 Standing waves patterns in (a) a tube open at both ends (a half-wavelength resonator) and (b) a tube open at one end and closed at the other end (a quarterwavelength resonator).
corresponds to exact halves, thirds, etc., of the tube length, as illustrated in Fig. 1.17a. In turn, these modes produce harmonics that are exact multiples of the fundamental frequency. Harmonics will occur at each multiple of the fundamental frequency for a tube open at both ends.

Air particles vibrating in a tube that is closed at one end and open at the other end are restricted most at the closed end. As a result, their displacement will be least at the closed end, where the pressure is the greatest. Thus, in terms of displacement, there must be a node at the closed end and an antinode at the open end, as illustrated in Fig. 1.17b. This pattern is analogous to the distance from a zero crossing to a peak, which corresponds to one quarter of a cycle ( $0^{\circ}$ to $90^{\circ}$ ), and a distance of one quarter of a wavelength ( $\lambda / 4$ ). Because the length of the tube corresponds to $\lambda / 4$, its lowest resonant frequency has a wavelength that is four times the length of the tube (4L). Hence, $f=c / 4 L$. For this reason, a tube that is open at one end and closed at the other end is a
quarter-wavelength resonator. Because a node can occur at only one end, these tubes have only odd modes of vibration and produce only odd harmonics of the fundamental frequency (e.g., $f 1, f 3, f 5, f 7$, etc.), as illustrated in the figure.

## Immittance

Immittance is the general term used to describe how well energy flows through a system. The opposition to the flow of energy is called impedance ( $Z$ ). The inverse of impedance is called admittance ( $\mathbf{Y}$ ), which is the ease with which energy flows through a system.

The concept of impedance may be understood in terms of the following example. (Although this example only considers mass, we will see that immittance actually involves several components.) Imagine two metal blocks weighing different amounts. Suppose you repetitively push and pull the lighter block back and forth across a smooth table top with a certain amount of effort. This is a mechanical system in which a sinusoidally alternating force (the pushing and pulling) is being applied to a mass (the block). The effort with which you are pushing (and pulling) the block is the amount of applied force, and the velocity of the block reflects how well energy flows through this system to effect motion. A particular block will move at a certain velocity given the amount of effort you are using to push (and pull) it. If the same amount of effort was used to push and pull the heavier block, then it would move slower than the first one. In other words, the heavier block (greater mass) moves with less velocity than the lighter block (smaller mass) in response to the same amount of applied force. We can say that the flow of energy is opposed more by the heavier block than by the lighter one. For this reason, the heavier block (greater mass) has more impedance and less admittance than the lighter block (smaller mass).

This example shows that impedance and admittance are viewed in terms of the relationship between an applied force and the resulting amount of velocity. In effect, higher impedance means that more force must be applied to result in a given amount of velocity, and lower impedance means that less force is needed to result in a given amount of velocity. For the mathematically oriented, we might say that impedance $(Z)$ is the ratio of force to velocity:

$$
Z=\frac{F}{v}
$$

The amount of impedance is expressed in ohms. The larger the number of ohms, the greater the opposition to the flow of energy.

Block: Mass Reactance ( $\mathrm{X}_{\mathrm{m}}$ )

Spring: Stiffness Reactance $\left(\mathrm{X}_{\mathrm{s}}\right)$

Fig. 1.18 The components of impedance are (1) mass reactance ( $X_{m}$ ), represented by the block; (2) stiffness reactance $\left(X_{s}\right)$, represented by the spring; and (3) resistance ( $R$ ), represented by the rough surface under the block.

Admittance $(Y)$ is the reciprocal of impedance:

$$
Y=\frac{1}{Z}
$$

and is therefore equal to the ratio of velocity to force:

$$
Y=\frac{v}{F}
$$

As we might expect, the unit of admittance is the inverse of the ohm, and is therefore called the mho. The more mhos, the greater the ease with which energy flows. The admittance values that we are concerned with in audiology are very small, and are thus expressed in millimhos (mmhos).

Impedance involves the complex interaction of three familiar physical components: mass, stiffness, and friction. In Fig. $\mathbf{1 . 1 8}$ mass is represented by the block, stiffness (or compliance) by the spring, and friction by the rough surface under the block. Let's briefly consider each of these components. Friction dissipates some of the energy being introduced into the system by converting it into heat. This component of impedance is called resistance $(R)$. The effect of resistance occurs in-phase with the applied force (Fig. 1.19). Some amount of friction is always present. Opposition to the flow of energy due to mass is called mass (positive) reactance $\left(\boldsymbol{X}_{\boldsymbol{m}}\right)$ and is related to inertia. The opposition due to the stiffness of a system is called stiffness (negative) reactance ( $X_{s}$ ), and is related to the restoring force that develops when an elastic element (e.g., a spring) is displaced.

Mass and stiffness act to oppose the applied force because these components are out-of-phase with it (Fig. 1.19). They oppose the flow of energy by storing it in these out-of-phase components before effecting motion. First, consider the mass all by itself. At the same point in time (labeled 1) the applied force is maximal (in the upward direction) and the velocity of the block (mass) is zero (crossing the horizontal line in the positive direction). One quarter of


Fig. 1.19 Relationship between a sinusoidally applied force (top) and the velocities associated with mass, stiffness, and resistance. The dotted lines labeled 1 and 2 show two moments in time. Resistance is in-phase with the applied force (F). Mass and stiffness are $90^{\circ}$ out-of-phase with force and $180^{\circ}$ out-of-phase with each other.
a cycle later, at the time labeled 2, the applied force is zero and the velocity of the mass is now maximal (in the upward direction). Hence, a sinusoidally applied force acting on a mass and the resulting velocity of the mass are a quarter-cycle $\left(90^{\circ}\right)$ out-ofphase. To appreciate this relationship, hold a weight and shake repetitively back and forth from right to left. You will feel that you must exert the most effort (maximal force) at the extreme right and left points of the swing, where the direction changes. Notice that the weight is momentarily still (i.e., its velocity is zero) at the extreme right and left points because this is where it changes direction. On the other hand, the weight will be moving the fastest (maximum velocity) as it passes the midpoint of the right-to-left
swing, which is also where you will be using the least effort (zero force).

Now consider the stiffness all by itself at the same two times in Fig. 1.19. At time 1, when the applied force is maximal (upward), the velocity of the spring (stiffness) is zero (crossing the horizontal line in the negative direction). One quarter-cycle later, at time 2, the applied force is zero, and the velocity of the spring is now maximal (downward). Hence, a sinusoidally applied force acting on a spring and the resulting velocity of the spring (stiffness) are a quarter-cycle ( $90^{\circ}$ ) out-of-phase. This occurs in the opposite direction of what we observed for the mass (whose motion is associated with inertia) because the motion of the spring (stiffness) is associated with restoring force. You can appreciate this relationship of the stiffness component by alternately compressing and expanding a spring. You must push or pull the hardest (maximum applied force) at the moment when the spring is maximally expanded (or compressed), which is also when the spring is not moving (zero velocity) because it is about to change direction. Similarly, you will exert no effort (zero applied force) and the spring will be moving the fastest (maximum velocity) as it moves back through its "normal" position (where it is neither compressed nor expanded).

Notice that the mass and stiffness reactances are $180^{\circ}$ out-of-phase with each other (Fig. 1.19). This means that the effects of mass reactance and stiffness reactance oppose each other. As a result, the net reactance ( $\boldsymbol{X}_{\text {net }}$ ) is the difference between them, so that

$$
X_{\text {net }}=X_{s}-X_{m}
$$

when stiffness reactance is larger, or

$$
X_{\text {net }}=X_{m}-X_{s}
$$

when mass reactance is larger. For example, if $X_{s}$ is 850 ohms and $X_{m}$ is 140 ohms, then $X_{\text {net }}$ will be $850-140=710$ ohms of stiffness reactance. If $X_{m}$ is 1000 ohms and $X_{s}$ is 885 ohms, then $X_{\text {net }}$ will be $1000-885=115$ ohms of mass reactance.

The overall impedance is obtained by combining the resistance and the net reactance. This cannot be done by simple addition because the resistance and reactance components are out-of-phase. (Recall here the difference between scalars and vectors mentioned at the beginning of the chapter.) The relationships in Fig. 1.20 show how impedance is derived from resistance and reactance. The size of the resistance component is plotted along the $x$-axis. Reactance is plotted on the $y$-axis, with mass (positive) reactance represented upward and stiffness (negative) reactance downward. The net reactance here is plotted downward because $X_{s}$ is greater


Fig. 1.20 Impedance $(Z)$ is the complex interaction of resistance $(R)$ and the net reactance ( $X_{\text {net }}$, which is equal to $\left.X_{s}-X_{m}\right)$. Notice the impedance value is determined by the vector addition of the resistance and reactance. The angle (9) between the horizontal leg of the triangle (resistance) and its hypotenuse (impedance) is called the phase angle.
than $X_{m}$, so that $X_{\text {net }}$ is negative. Notice that $R$ and $X_{\text {net }}$ form two legs of a right triangle, and that $Z$ is the hypotenuse. Hence, we find $Z$ by the familiar Pythagorean theorem $\left(a^{2}+b^{2}=c^{2}\right)$, which becomes $Z^{2}=R^{2}+X_{\text {net }}^{2}$. Removing the squares gives us the formula for calculating impedance from the resistance and reactance components:

$$
Z=\sqrt{R^{2}+X_{\text {net }}^{2}}
$$

Resistance tends to be essentially the same at all frequencies. However, reactance depends on frequency $(f)$ in the following way: (1) mass reactance is proportional to frequency,

$$
X_{m}=2 \pi f M
$$

where $M$ is mass; and (2) stiffness reactance is inversely proportional to frequency,

$$
X_{s}=\frac{S}{2 \pi f}
$$

where $S$ is stiffness. In other words, $X_{m}$ gets larger as frequency goes up, and $X_{s}$ gets larger as frequency goes down. Because of these frequency relationships, impedance also depends on frequency:

$$
Z=\sqrt{R^{2}+\left(\frac{s}{2 \pi f}+2 \pi f M\right)^{2}}
$$

In addition, there will be a frequency where $X_{m}$ and $X_{s}$ are equal, and thus cancel. This is the resonant frequency, where the only component that is opposing the flow of energy is resistance.

Admittance is the reciprocal of impedance,

$$
Y=\frac{1}{Z}
$$

and the components of admittance are the reciprocals of resistance and reactance: conductance $(G)$ is the reciprocal of resistance,

$$
G=\frac{1}{R}
$$

stiffness (compliance) susceptance ( $\boldsymbol{B}_{s}$ ) is the reciprocal of stiffness reactance,

$$
B_{s}=\frac{1}{X_{s}}
$$

and mass susceptance $\left(\boldsymbol{B}_{\boldsymbol{m}}\right)$ is the reciprocal of mass reactance:

$$
B_{m}=\frac{1}{X_{m}}
$$

Stiffness susceptance is proportional to frequency ( $B_{s}$ increases as frequency goes up), and mass susceptance is inversely proportional to frequency ( $B_{m}$ decreases as frequency goes up). Net susceptance ( $B_{\text {net }}$ ) is the difference between $B_{s}$ and $B_{m}$. The formula for admittance is

$$
Y=\sqrt{G^{2}+B_{\text {net }}^{2}}
$$

where $B_{\text {net }}$ is $\left(B_{s}-B_{m}\right)$ when $B_{s}$ is bigger and ( $B_{m}-$ $B_{s}$ ) when $B_{m}$ is larger.

Up to this point we have discussed immittance in mechanical terms. Acoustic immittance is the term used for the analogous concepts when dealing with sound. The opposition to the flow of sound energy is called acoustic impedance $\left(Z_{a}\right)$, and its reciprocal is acoustic admittance $\left(\boldsymbol{Y}_{a}\right)$. Thus,

$$
Z_{a}=\frac{1}{Y_{a}}
$$

and

$$
Y_{a}=\frac{1}{Z_{a}}
$$

When dealing with acoustic immittance, we use sound pressure $(\boldsymbol{p})$ in place of force, and velocity is replaced with the velocity of sound flow, called volume velocity $(\boldsymbol{U})$. Thus, acoustic impedance is simply the ratio of sound pressure to volume velocity,

$$
Z_{a}=\frac{p}{U}
$$

and acoustic admittance is the ratio of volume velocity to sound pressure,

$$
Y_{a}=\frac{U}{p}
$$

The components of acoustic immittance are based on the acoustic analogies of friction, mass, and stiffness (compliance). Friction develops between air molecules and a mesh screen, which is thus used to model acoustic resistance ( $\boldsymbol{R}_{a}$ ). Mass (positive) acoustic reactance $\left(+\boldsymbol{X}_{\boldsymbol{a}}\right)$ is represented by a slug of air in an open tube. Here an applied sound pressure will displace the slug of air as a unit, so that its inertia comes into play. A column of air inside a tube open at one end and closed at the other end represents stiffness (negative) acoustic reactance ( $-\boldsymbol{X}_{a}$ ) because sound pressure compresses the air column like a spring. The formulas and relationships for acoustic immittance are the same as those previously given, except that the analogous acoustic values are used. For example, acoustic impedance is equal to

$$
Z_{a}=\sqrt{R_{a}^{2}+X_{a}^{2}}
$$

where $X_{a}$ is the net difference between stiffness acoustic reactance ( $-X_{a}$ ) and mass acoustic reactance $\left(+X_{a}\right)$. Similarly, the formula for acoustic admittance is

$$
Y_{a}=\sqrt{G_{a}^{2}+B_{a}^{2}}
$$

where $B_{a}$ is the net difference between stiffness acoustic susceptance ( $+B_{a}$ ) and mass acoustic susceptance $\left(-B_{a}\right)$.

## Expressing Values in Decibels

It is extremely cumbersome to express sound magnitudes in terms of their actual intensities or pressures for several reasons. To do so would involve working in units of watts $/ \mathrm{m}^{2}$ (or watts $/ \mathrm{cm}^{2}$ ) and newtons $/ \mathrm{m}^{2}$ (or dynes $/ \mathrm{cm}^{2}$ ). In addition, the range of sound magnitudes with which we are concerned in audiology is enormous; the loudest sound that can be tolerated has a pressure that is roughly 10 million times larger than the softest sound that can be heard. Even if we wanted to work with such an immense range of cumbersome values on a linear scale, we would find that it is hard to deal with them in a way that has relevance to the way we hear. As a result, these absolute physical values are converted into a simpler and more convenient form called decibels (dB) to make them palatable and meaningful.

The decibel takes advantage of ratios and logarithms. Ratios are used so that physical magnitudes can be stated in relation to a reference value that has meaning to us. It makes sense to use the softest sound that can be heard by normal people as our reference value. This reference value has an intensity of

$$
10^{-12} \mathrm{~W} / \mathrm{m}^{2}
$$

in MKS units, which corresponds to

$$
10^{-16} \mathrm{~W} / \mathrm{cm}^{2}
$$

in the cgs system. The same softest audible sound can also be quantified in terms of its sound pressure. This reference pressure is

$$
2 \times 10^{-5} \mathrm{~N} / \mathrm{m}^{2}
$$

or

$$
20 \mu \mathrm{~Pa}
$$

in the MKS system. ${ }^{2}$ In cgs units this reference pressure is

$$
2 \times 10^{-4} \text { dyne } / \mathrm{cm}^{2}
$$

(The student will find that $2 \times 10^{-4}$ dyne $/ \mathrm{cm}^{2}$ is also written as 0.0002 dynes $/ \mathrm{cm}^{2}, 2 \times 10^{-4} \mu$ bar, or 0.0002 $\mu \mathrm{bar}$, especially in older literature.) The appropriate reference value (intensity or pressure, MKS or cgs) becomes the denominator of our ratio, and the intensity (or pressure) of the sound that is actually being measured or described becomes the numerator. As a result, instead of describing a sound that has an intensity of $10^{-10} \mathrm{~W} / \mathrm{m}^{2}$, we place this value into a ratio so we can express it in terms of how it compares to our reference value (which is $10^{-12}$ $\mathrm{W} / \mathrm{m}^{2}$ ). Hence, this ratio would be

$$
\frac{10^{-10} \mathrm{~W} / \mathrm{m}^{2}}{10^{-12} \mathrm{~W} / \mathrm{m}^{2}}
$$

This ratio reduces to simply $10^{2}$, or 100 .
Regardless of what this ratio turns out to be, it is replaced with its common logarithm because equal ratios correspond to different distances on a linear scale, but equal ratios correspond to equal distances on a logarithmic scale. In other words, the linear distance between two numbers with the same ratio relationship (e.g., 2 to 1 ) is small for small numbers and large for large numbers, but the logarithm of that ratio is always the same. For example, all of the following pairs involve $2 / 1$ ratios. Even though the linear distance between the numbers in the pairs gets wider as the absolute sizes of the numbers get larger, the logarithm of all of the ratios stays the same ( $2 / 1=2$, and $\log 2$ is always 0.3 ; Table 1.4).

The general decibel formula is expressed in terms of power as follows:

$$
P L=10 \log \frac{\mathrm{P}}{\mathrm{P}_{0}}
$$

Here, $P L$ stands for power level (in dB), $P$ is the power of the sound being measured, and $P_{0}$ is the reference power to which the former is being compared. The word level is added to distinguish the raw physical

[^0]Table 1.4 All pairs of numbers that have the same ratio between them (e.g., 2:1) are separated by the same logarithmic distance even though their linear distances are different

| Pairs of <br> numbers <br> with 2:1 ratios | Distances <br> between the <br> absolute numbers <br> get wider | Logarithms of <br> all 2:1 ratios <br> are the same |
| :--- | :--- | :--- |
| $2 / 1$ | 1 | $\mathbf{0 . 3}$ |
| $8 / 4$ | 4 | $\mathbf{0 . 3}$ |
| $20 / 10$ | 10 | $\mathbf{0 . 3}$ |
| $100 / 50$ | 50 | $\mathbf{0 . 3}$ |
| 200/100 | 100 | $\mathbf{0 . 3}$ |
| 2000/1000 | 1000 | $\mathbf{0 . 3}$ |

quantity (power) from the corresponding decibel value (which is a logarithmic ratio about the power). Similarly, intensity expressed in decibels is called intensity level (IL) and sound pressure expressed in decibels is called sound pressure level (SPL).

Most sound measurements are expressed in terms of intensity or sound pressure, with the latter being the most common. The formula for decibels of intensity level is

$$
I L=10 \log \frac{I}{I_{0}}
$$

where $I L$ is intensity level in $\mathrm{dB}, I$ is the intensity of the sound in question (in W/m²), and $I_{0}$ is the reference intensity $\left(10^{-12} \mathrm{~W} / \mathrm{m}^{2}\right)$. If the value of $I$ is $10^{-10} \mathrm{~W} / \mathrm{m}^{2}$, then

$$
\begin{aligned}
I L & =10 \log \frac{10^{-10} \mathrm{~W} / \mathrm{m}^{2}}{10^{-12} \mathrm{~W} / \mathrm{m}^{2}} \\
& =10 \log \frac{10^{-10}}{10^{-12}}\left(\text { notice that } \mathrm{W} / \mathrm{m}^{2}\right. \text { cancels out) } \\
& =10 \log 10^{(-10)-(-12)} \\
& =10 \log 10^{2} \\
& =10 \times 2 \\
& =20 \mathrm{~dB} \mathrm{re}: 10^{-12} \mathrm{~W} / \mathrm{m}^{2} \\
& =20 \mathrm{~dB} \mathrm{IL}
\end{aligned}
$$

Consequently, an absolute intensity of $10^{-10} \mathrm{~W} / \mathrm{m}^{2}$ has an intensity level of 20 dB re: $10^{-12} \mathrm{~W} / \mathrm{m}^{2}$, or 20 dB IL. The phrase "re: $10^{-12} \mathrm{~W} / \mathrm{m}^{2}$ " is added
because the decibel is a dimensionless quantity that has real meaning only when we know the reference value, that is, the denominator of the ratio.

The formula for decibels of sound pressure level (dB SPL) is obtained by replacing all of the intensity values with the corresponding values of pressure squared (because $I \propto p^{2}$ ):

$$
S P L=10 \log \frac{p^{2}}{p_{0}^{2}}
$$

Here, $p$ is the measured sound pressure (in $\mathrm{N} / \mathrm{m}^{2}$ ) and $p_{0}$ is the reference sound pressure $\left(2 \times 10^{-5} \mathrm{~N} / \mathrm{m}^{2}\right.$, or $20 \mu \mathrm{~Pa}$ ). This form of the formula is cumbersome because of the squared values, which can be removed by applying the following steps:

$$
\begin{aligned}
& S P L=10 \log \frac{p^{2}}{p_{0}^{2}} \\
& S P L=10 \log \left(\frac{p}{p}\right)^{2}\left(\text { because } \frac{x^{2}}{y^{2}}=\left(\frac{x}{y}\right)^{2}\right) \\
& S P L=10 \times 2 \log \left(\frac{p}{p_{0}}\right) \quad\left(\text { because } x^{2}=2 \log x\right) \\
& S P L=20 \log \left(\frac{p}{p_{0}}\right)
\end{aligned}
$$

Therefore, the commonly used simplified formula for decibels of SPL is

$$
S P L=20 \log \frac{\mathrm{p}}{\mathrm{p}_{0}}
$$

where the multiplier is 20 instead of 10 as a result of removing the squares from the unsimplified version of the formula.

Let us go through the exercise of converting the absolute sound pressure of a sound into dB SPL. We will assume that the sound being measured has a pressure of $2 \times 10^{-4} \mathrm{~N} / \mathrm{m}^{2}$. Recall that the reference pressure is $2 \times 10^{-5} \mathrm{~N} / \mathrm{m}^{2}$. The steps are as follows:

$$
\begin{aligned}
S P L & =20 \log \frac{2 \times 10^{-4} \mathrm{~N} / \mathrm{m}^{2}}{2 \times 10^{-5} \mathrm{~N} / \mathrm{m}^{2}} \\
& =20 \log \frac{10^{-4}}{10^{-5}}\left(\text { notice that } \mathrm{N} / \mathrm{m}^{2} \text { cancels out }\right) \\
& =20 \log 10^{(-4)-(-5)} \\
& =20 \log 10^{1} \\
& =20 \times 1=20 \\
& =20 \mathrm{~dB} \text { re }: 2 \times 10^{-5} \mathrm{~N} / \mathrm{m}(\text { or } 20 \mu \mathrm{~Pa}) \\
& =20 \mathrm{~dB} \text { SPL }
\end{aligned}
$$

Hence, a sound pressure of $2 \times 10^{-4} \mathrm{~N} / \mathrm{m}^{2}$ corresponds to a sound pressure level of 20 dB re: $2 \times 10^{-5}$ $\mathrm{N} / \mathrm{m}^{2}$ ( or $20 \mu \mathrm{~Pa}$ ), or 20 dB SPL.

What is the decibel value of the reference itself? In other words, what would happen if the intensity (or pressure) being measured is equal to the reference intensity (or pressure)? In terms of intensity, the answer is found by using the reference value $\left(10^{-12} \mathrm{w} / \mathrm{m}^{2}\right)$ as both the numerator $(I)$ and denominator $\left(I_{0}\right)$ in the dB formula, so that

$$
\begin{aligned}
I L & =10 \log \frac{10^{-12} \mathrm{~W} / \mathrm{m}^{2}}{10^{-12} \mathrm{~W} / \mathrm{m}^{2}} \\
& =10 \log 1(\text { anything divided by itself equals } 1) \\
& =10 \times 0 \\
& =0 \mathrm{~dB} \text { re: } 10^{-12} \mathrm{~W} / \mathrm{m}^{2} \\
& =0 \mathrm{~dB} \mathrm{IL}
\end{aligned}
$$

Consequently, the intensity level of the reference intensity is 0 dB IL. Similarly, 0 dB SPL means that the measured sound pressure corresponds to that of the reference sound:

$$
\begin{aligned}
S P L & =20 \log \frac{2 \times 10^{-5} \mathrm{~N} / \mathrm{m}^{2}}{2 \times 10^{-5} \mathrm{~N} / \mathrm{m}^{2}} \\
& =20 \log 1 \text { (anything divided by itself equals } 1) \\
& =20 \times 0 \\
& =0 \mathrm{~dB} \text { re }: 2 \times 10^{-5} \mathrm{~N} / \mathrm{m}^{2}(\text { or } 20 \mu \mathrm{~Pa}) \\
& =0 \mathrm{~dB} \mathrm{SPL}
\end{aligned}
$$

Notice that 0 dB IL or 0 dB SPL means that the sound being measured is equal to the reference value; it does not mean "no sound." It follows that negative decibel values indicate that the magnitude of the sound is lower than the reference; for example, -10 dB means that the sound in question is 10 dB below the reference value.

## Sound Measurement

The magnitude of a sound is usually measured with a device called a sound level meter (SLM). This device has a high-quality microphone that picks up the sound and converts it into an electrical signal that is analyzed by an electronic circuit, and then displays the magnitude of the sound on a meter in decibels of sound pressure level ( dB SPL). An example of an SLM is shown in Fig. 1.21. SLMs are used to calibrate or establish the accuracy of audiometers and other instruments used to test hearing, as well as to measure noise


Fig. 1.21 An example of a digital sound level meter with a measuring microphone attached.
levels for such varied purposes as determining whether a room is quiet enough for performing hearing tests or identifying potentially hazardous noise exposures. SLMs or equivalent circuits that perform the same function are also found as components of other devices, such as hearing aid test systems.

The characteristics of SLMs are specified in the ANSI S1.4-2014 (R2019) standard. The accuracy of the measurements produced by an SLM is established using a compatible acoustical calibrator, which is a device that produces a known precise signal that is directed into the SLM microphone. Fig. 1.22 shows an example of one type of acoustical calibrator, known as a piston phone because of the way it works. A barometer is shown to the right of the piston phone, which is needed because these measurements are affected by barometric pressure. For example, if the calibrator produces a signal that is exactly 114 dB SPL, then the SLM is expected to read this amount when it is connected to the calibrator (within certain tolerances allowed in the ANSI standard). If the meter reading deviates from the actual value of 114 dB SPL, then its controls are adjusted to reset the meter to the right value, or it might be necessary to have the SLM repaired and recalibrated by the manufacturer or an instrumentation service company.


Fig. 1.22 An example of a piston phone acoustical calibrator (a), an analog barometer (b), and a digital barometer (c). (Photographs courtesy of GRAS Sound and Vibration.)

The microphone of an SLM picks up all sounds that are present at all frequencies within its operating range. However, the user might want to emphasize or de-emphasize certain frequency ranges (we'll talk about why later). The SLM has settings to make these adjustments or corrections, which are called weightings. In other words, the SLM has weightings that can treat the original sound levels as though they were stronger or weaker at certain frequencies. For example, imagine that the actual level picked up by the microphone at, say, 700 Hz is 49 dB . If the SLM has a weighting of -11 dB at that frequency, then it will be treated as though it was $49-11=38 \mathrm{~dB}$. If the weighting is +2 dB , then it would be treated as though it was $49+2=51 \mathrm{~dB}$. One of these weightings is actually no weighting at all, or unweighted, so that the sound levels at all frequencies are treated exactly as they were picked up by the microphone. In this case, we could say that the weighting is zero decibels. This setting on the SLM is therefore called $Z$ for zero, as in zero adjustments or a zero weighting. This setting is therefore the $Z$-weighting (sometimes called the $Z$ scale). So, when the SLM is set to the $Z$-weighting it will display the overall SPL based on all of the sounds picked up by the microphone without making any adjustments or corrections. In this case, the


Fig. 1.23 Frequency response curves for the $A-, B-, C$, and $Z$-weightings (see text).
sound level is expressed in dB SPL or dBZ. (Some older SLMs called this the "linear" setting.)

SLMs also have other weightings that do change the emphasis given to certain parts of the spectrum, and these are used much more often than Z. In fact, the ANSI standard for SLMs actually considers the Zweighting optional. We all know that turning up the bass on a radio, TV, or music system makes the low pitches more pronounced, whereas turning the bass down makes the lows less noticeable. In other words, the bass control determines whether the low frequencies will be emphasized or de-emphasized. The treble control does the same thing for the high frequencies. SLM weightings do essentially the same thing, mainly by de-emphasizing the low frequencies.

Fig. 1.23 shows several of the weightings that may be found on SLMs, of which $\mathbf{A}$ and $\mathbf{C}$ are the most common. The y -axis is relative level in decibels, and shows how many decibels are subtracted or added by the weighting at a given frequency. A horizontal line at 0 dB refers to how the sound would be without the weighting network. In other words, 0 dB here means "unchanged" or " 0 dB of change," and corresponds to the Z-weighting as described above. Almost all of the adjustments are negative, and show how much the sound level is de-emphasized at each frequency.

The A-weighting considerably de-emphasizes the low frequencies, as shown by its curve, which gets progressively more negative as frequency decreases below 1000 Hz . For example, the curve shows that the A-weighting network de-emphasizes sounds by $\sim 4 \mathrm{~dB}$ at $500 \mathrm{~Hz}, 11 \mathrm{~dB}$ at $200 \mathrm{~Hz}, 19 \mathrm{~dB}$ at 100 Hz , and 30 dB at 50 Hz . This is analogous to turning the bass all the way down on a stereo system. Sound level measurements using the Aweighting are expressed as dBA. The A-weighting is commonly used when it is desirable to exclude the effects of the lower frequencies, and are especially useful in noise level measurements.


Fig. 1.24 A series of octave-bands representing those typically used in octave-band analysis. Notice the bands overlap at their 3 dB down points. The 1000 Hz octaveband is highlighted for clarity.

The B-weighting network also de-emphasizes the lower frequencies, but not as much as the Aweighting. For example, the amount of reduction is only $\sim 6 \mathrm{~dB}$ at 100 Hz . Sound level measurements using the B-weighting are expressed as dBB. However, the B-weighting is rarely used and no longer included in the ANSI standard for SLMs.

The C-weighting is barely different from a flat, unweighted $(Z)$ response. Sound level measurements using the C-weighting are expressed as $\mathbf{d B C}$. The Cweighting is commonly used as a proxy for the completely flat $(Z)$ response, and useful in noise level measurements.

In addition to frequency weightings, SLMs often have filters that allow it to "look at" a certain range of frequencies instead of all of them. Octave-band filters separate the overall frequency range into narrower ranges, each of which is one octave wide, as illustrated in Fig. 1.24. For example, the range from 355 to 710 Hz is an octave-band because $710=2 \times 355$, and the bandwidth from 2800 to 5600 Hz is also an octave-band because $5600=2 \times 2800$. An octaveband is named according to its center frequency, but keep in mind that the center is defined as the geometric mean of the upper and lower cutoffs rather than the arithmetic midpoint between them. Hence, the 500 Hz octave-band goes from approximately 355 to 710 Hz , and the 4000 Hz octave-band includes 2800 to 5600 Hz . The center frequencies and the upper and lower cutoff frequencies of the octave-bands typically used in acoustical measurements are listed in Table 1.5.

Measuring a noise on an octave-band by octave-band basis is called octave-band analysis and makes it possible to learn about the spectrum of a sound instead of just its overall level. An even finer level of analysis can be achieved by using third-octave-band filters, in which case each filter is one-third of an octave wide. For example, the 500 Hz third-octave filter includes the frequencies between approximately 450 and 560 Hz , and the 4000 Hz third-octave-band goes from 3550 to

Table 1.5 Examples of octave-band center frequencies, and lower and upper cutoff frequencies

| Center <br> frequency (Hz) | Lower cutoff <br> $(\mathrm{Hz})$ | Upper cutoff <br> $(\mathrm{Hz})$ |
| :--- | :--- | :--- |
| 31.5 | 22.4 | 45 |
| 63 | 45 | 90 |
| 125 | 90 | 180 |
| 250 | 180 | 355 |
| 500 | 355 | 710 |
| 1000 | 710 | 1400 |
| 2000 | 1400 | 2800 |
| 4000 | 2800 | 5600 |
| 8000 | 5600 | 11,200 |
| 16,000 | 11,200 | 22,630 |

4500 Hz . Octave-band and third-octave-band filters are useful when we want to concentrate on the sound level in a narrow frequency range without contamination from other frequencies. For example, it is usually better to measure the level of a 1000 Hz tone while using a filter centered around 1000 Hz than to do the same thing with the all-inclusive $Z$ setting because the filter excludes other frequencies that would contaminate the results.

We can combine octave-band levels (OBLs) or third-octave-band levels $(1 / 3$-OBLs) to arrive at the overall level of a sound. There are two ways to combine OBLs into overall SPL. The simpler approach involves adding the OBLs in successive pairs using the rules for adding decibels shown in Table 1.6. It is very easy to use this table. First, find the difference in decibels between the two sounds being combined. For example, if one sound is 80 dB and the other is 76 dB , then the difference between them is 4 dB . Then find the increment that corresponds to the difference. According to the table, the increment for a 4 dB difference is 1.4 dB . Now, just add this increment to the larger of the original two sounds. The larger value in our example is 80 dB , so we add the 1.4 dB increment to $80 \mathrm{~dB}(80+1.4=81.4)$. Hence, combining 80 and 76 dB results in a total of 81.4 dB . To combine octave-bands into an overall level, simply arrange their OBLs from largest to smallest, and combine pairs successively using the increments in the table. A complete example is shown in Appendix $A$.

Table 1.6 Combining decibels: find the difference in decibels between the two sounds, and then add the corresponding decibel increment to the larger original decibel value

| Difference in dB between <br> original sounds | Increment in dB (add to <br> larger original sound) |
| :--- | :--- |
| 0 | 3.0 |
| 1 | 2.6 |
| 2 | 2.2 |
| 3 | 1.8 |
| 4 | 1.4 |
| 5 | 1.2 |
| 6 | 1.0 |
| 7 | 0.8 |
| 8 | 0.6 |
| 9 | 0.5 |
| 10 | 0.4 |
| 11 | 0.35 |
| 12 | 0.3 |
| 13 | 0.25 |
| 14 | 0.2 |
| 15 | 0.15 |
| 16 | 0.1 |

The more precise method for combining octaveband levels into an overall SPL is to use the following formula for logarithmic addition:

$$
L=10 \log \sum_{\mathrm{i}=1}^{\mathrm{n}} 10^{\mathrm{Li} / 10}
$$

In this formula, $L$ is the overall (combined) level in dB SPL, $n$ is the number of bands being combined, $i$ is the $i$ th band, and $L_{i}$ is the OBL of the $i$ th band. An example showing how this formula is used may be found in Appendix A.

The same methods can be used to combine octave-band levels into an A-weighted sound level (dBA), except that a correction factor is applied to each OBL. This correction factor is the amount by which the A-weighting de-emphasizes the level of the sounds within each octave-band. Table 1.7 shows the corrections (dBA weightings) that can be used to convert unweighted octave-band levels into

Table 1.7 Corrections (dBA weightings) to convert unweighted octave-band levels into A-weighted octave-band levels

| Octave-band center |  |
| :--- | :--- |
| Frequency $(\mathrm{Hz})$ | dBA weighting |
| 31.5 | -39.4 |
| 63 | -26.2 |
| 125 | -16.1 |
| 250 | -8.6 |
| 500 | -3.2 |
| 1000 | 0 |
| 2000 | +1.2 |
| 4000 | +1.0 |
| 8000 | -1.1 |

A-weighted octave-band levels. For example, dBA deemphasizes the 125 Hz octave-band by 16.1 dB . Thus, if the 125 Hz OBL is 60 dB , we correct it to its dBA value by subtracting: $60-16.1=43.9 \mathrm{dBA}$. A full example is shown in Appendix A. The formula for more precisely converting OBLs into dBA is as follows:

$$
L_{A}=10 \log \sum_{i=1}^{n} 10^{\left(L_{i}+k_{i}\right) / 10}
$$

The symbols here are the same as in the previous formula except $L_{A}$ is now the overall (combined) level in dBA, and $k_{i}$ is the correction factor that must be applied to the OBL of the $i$ th band to convert it into its equivalent value in CBA (which is the reason for the term $\left(L_{i}+k_{i}\right)$ in the equation). Appendix A shows an example of how this formula is used.

## Study Questions

1. Define and specify the units of measurement for the following terms: displacement, velocity, acceleration, force, work, and power.
2. Explain pressure and intensity, state the reference values for each of them, and explain why we have these reference values.
3. Explain what happens to the intensity of a sound with increasing distance from the sound source.
4. Define simple harmonic motion and explain how its characteristics are depicted by a sine wave.
5. Define the terms cycle, period, frequency, and wavelength, and explain how they are related to each other.
6. Define friction and explain why it causes damping.
7. Define complex periodic and aperiodic waves, and describe how their characteristics are shown on the waveform and spectrum.
8. What are resonant frequencies and how are they related to standing waves?
9. Define impedance and describe how it is related to mass and stiffness.
10. What are the formulas and reference values for intensity level and sound pressure level in decibels? Explain how the magnitude of a sound is expressed in decibels.

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# Anatomy and Physiology of the Auditory System 

## General Overview

Hearing and its disorders are intimately intertwined with the anatomy and physiology of the auditory system, which is composed of the ear and its associated neurological pathways. The auditory system is fascinating, but learning about it for the first time means that we must face many new terms, relationships, and concepts. For this reason it is best to begin with a general bird's-eye view of how the ear is set up and how a sound is converted from vibrations in the air to a signal that can be interpreted by the brain. A set of self-explanatory drawings illustrating commonly used anatomical orientations and directions is provided in Fig. 2.1 for ready reference.

The major parts of the ear are shown in Fig. 2.3. One cannot help but notice that the externally visible auricle, or pinna, and the ear canal (external auditory meatus) ending at the eardrum (tympanic membrane) make up only a small part of the overall auditory system. This system is divided into several main sections: The outer ear includes the pinna and ear canal. The air-filled cavity behind the eardrum is called the middle ear, also known as the tympanic cavity. Fig. 2.2 shows how the structures of the hearing system are oriented within the head. Notice that the middle ear connects to the pharynx by the Eustachian tube. Medial to the middle ear is the inner ear. Three tiny bones (malleus, incus, and stapes), known as the ossicular chain, act as a bridge from the eardrum to the oval window, which is the entrance to the inner ear (Fig. 2.3).

The inner ear contains the sensory organs of hearing and balance. Our main interest is with the structures and functions of the hearing mechanism. Structurally, the inner ear is composed of the vestibule, which lies on the medial side of the oval window; the snail-shaped cochlea anteriorly;
and the three semicircular canals posteriorly. The entire system may be envisioned as a complex configuration of fluid-filled tunnels or ducts in the temporal bone, which is descriptively called the labyrinth. The labyrinth, which courses through the temporal bone, contains a continuous membranous duct within it, so that the overall system is arranged as a duct within a duct. The outer duct contains one kind of fluid (perilymph) and the inner duct contains another kind of fluid (endolymph). The part of the inner ear concerned with hearing is the cochlea. It contains the organ of Corti, which in turn has hair cells that are the actual sensory receptors for hearing. The balance (vestibular) system is composed of the semicircular canals and two structures contained within the vestibule, called the utricle and saccule.

The sensory receptor cells are in contact with nerve cells (neurons) that make up the eighth cranial (vestibuloacoustic) nerve, which connects the peripheral ear to the central nervous system. The auditory branch of the eighth nerve is often called the auditory or cochlear nerve, and the vestibular branches are frequently referred to as the vestibular nerve. The eighth nerve leaves the inner ear through an opening on the medial side of the temporal bone called the internal auditory meatus (canal), and then enters the brainstem. Here, the auditory portions of the nerve go to the cochlear nuclei and the vestibular parts of the nerve go to the vestibular nuclei.

The hearing process involves the following series of events. Sounds entering the ear set the tympanic membrane into vibration. These vibrations are conveyed by the ossicular chain to the oval window. Here, the vibratory motion of the ossicles is transmitted to the fluids of the cochlea, which in turn stimulate the sensory receptor (hair) cells of the organ of Corti. When the hair cells respond, they activate the neurons of the auditory nerve.


Fig. 2.1 (a-c) Commonly encountered anatomical planes, orientations, and directions.


Fig. 2.2 The auditory system in relation to the brain and skull. (Courtesy of Abbott Laboratories.)


Fig. 2.3 The major parts of the peripheral ear.

The signal is now in the form of a neural code that can be processed by the nervous system.

The outer ear and middle ear are collectively called the conductive system because their most apparent function is to bring (conduct) the sound signal from the air to the inner ear. The cochlea and eighth cranial nerve compose the sensorineural system, so named because it involves the physiological response to the stimulus, activation of the associated nerve cells, and the encoding of the sensory response into a neural signal. The aspect of the central nervous system that deals with this neurally encoded message is generally called the central auditory nervous system.

## Temporal Bone

To be meaningful, a study of the ear must begin with a study of the temporal bone. Most of the structures that make up the ear are contained within the temporal bone (Fig. 2.2). In fact, the walls of these structures and all of the bony aspects of the ear, except for the ossicles, are actually parts of the temporal bone itself. Recall from your anatomy class that the skeleton of the head is composed of 8 cranial bones and 14 facial bones. The right and left temporal bones compose the inferior lateral aspects of the cranium. Beginning posteriorly and moving clockwise, the temporal bone articulates with the occipital bone behind, the parietal bone behind and above, the sphenoid and zygomatic bones to the front, and the mandible anteriorly below. All of these connections, except for the articulation with the mandible, are firmly united, seam-like fibrous junctions called sutures. The articulation with the mandible is via the highly mobile temporomandibular joint.

Lateral and medial views of the temporal bone are shown in Fig. 2.4. The lateral surface of the bone faces the outside of the head and the medial surface
faces the inside of the head. The temporal bone is composed of five sections, including the mastoid, petrous, squamous, and tympanic parts, and the styloid process.

The squamous part is a very thin, fan-shaped portion on the lateral aspect of the bone. It articulates with the parietal bone posteriorly and superiorly, and with the sphenoid bone anteriorly. The prominent zygomatic process runs anteriorly to join with the zygomatic bone, forming the zygomatic arch on the medial side of the temporal bone. Just below the base of the zygomatic process is a depression called the mandibular fossa, which accepts the condyle of the mandible to form the temporomandibular joint just anterior to the ear canal.

The petrous part is pyramid-shaped and medially oriented so that it forms part of the base of the cranium. This extremely hard bone contains the inner ear and the internal auditory meatus through which the eighth cranial nerve travels on its way to the brainstem, so that much of the discussion pertaining to the inner ear is also a discussion of this part of the temporal bone.

The mastoid part composes the posterior portion of the temporal bone. It extends posteriorly from the petrous part, below and behind the squamous part. The mastoid articulates with the occipital bone posteriorly and with the parietal bone superiorly. It has an inferiorly oriented, cone-shaped projection below the skull base called mastoid process. The mastoid contains an intricate system of interconnecting air cells that vary widely in size, shape, and number. These are connected with an anterosuperior cavity called the tympanic antrum, which is located just behind the middle ear cavity. An opening called the aditus ad antrum connects the antrum with the attic or upper part of the middle ear cavity. The roof of the antrum (and the middle ear) is composed of a thin bony plate called the tegmen


Fig. 2.4 (a) Medial and (b) lateral views of the right temporal bone. (Adapted from Proctor 1989, with permission.)
tympani, which separates them from the part of the brain cavity known as the middle cranial fossa. Its medial wall separates it from the lateral semicircular canal of the inner ear. The middle ear, antrum, and air cells compose a continuous, air-filled system. Hence, it is not hard to imagine how an untreated middle ear infection can spread to the mastoid air cell system and beyond.

The tympanic part is inferior to the squamous and petrous parts and anterior to the mastoid. The tympanic part forms the inferior and anterior walls of the ear canal, as well as part of its posterior wall.

The styloid process is an anteroinferior pillarlike projection from the base of the temporal bone that varies widely in size. It does not contribute to the auditory structures but is of interest to us as the origin of several muscles involved in the speech mechanism.

## - Outer and Middle Ear

The outer ear is composed of the pinna and the ear canal, ending at the eardrum. The tympanic membrane is generally considered to be part of the middle ear system, which includes the middle ear cavity and its contents, and "ends" where the ossicles transmit the signal to the inner ear fluids at the oval window.

## Pinna

The externally visible aspect of the ear is an oddshaped appendage called the pinna or auricle. The internal structure of the pinna is composed principally of elastic cartilage (except for the earlobe).


Fig. 2.5 Major landmarks on the pinna.
It also contains some undifferentiated intrinsic muscle tissue, as well as several extrinsic muscles, although these are vestigial structures in humans. However, these muscles are not vestigial in many lower animals that are able to orient their pinnae with respect to the location of a sound source.

The major landmarks of the pinna are highlighted in Fig. 2.5. Notice that the pinna is not symmetrical. For example, and most obviously, it has a flap-like extension that angles away from the skull in the backward direction, so that the pinna overlaps the side of the head posteriorly, superiorly, and inferiorly,


[^0]:    ${ }^{2}$ One pascal (Pa) $=1 \mathrm{~N} / \mathrm{m}^{2}$. Thus, $10^{-6} \mathrm{~N} / \mathrm{m}^{2}=1$ micropascal $(\mu \mathrm{Pa}), 10^{-5}$ $\mathrm{N} / \mathrm{m}^{2}=10 \mu \mathrm{~Pa}$, and $2 \times 10^{-5} \mathrm{~N} / \mathrm{m}^{2}=20 \mu \mathrm{~Pa}$.

