CHAPTER 1

Introduction to Biomechanical Analysis

ANDREW R. KARDUNA, PH.D.

CHAPTER CONTENTS

Although the human body is an incredibly complex biological system composed of trillions of cells, it is subject to the same fundamental laws of mechanics that govern simple metal or plastic structures. The study of the response of biological systems to mechanical forces is referred to as **biomechanics**. Although it wasn't recognized as a formal discipline until the 20th century, biomechanics has been studied by the likes of Leonardo da Vinci, Galileo Galilei, and Aristotle. The application of biomechanics to the musculoskeletal system has led to a better understanding of both joint function and dysfunction, resulting in design improvements in devices such as joint arthroplasty systems and orthotic devices. Additionally, basic musculoskeletal biomechanics concepts are important for clinicians such as orthopedic surgeons and physical and occupational therapists.

Biomechanics is often referred to as the link between structure and function. While a therapist typically evaluates a patient from a kinesiologic perspective, it is often not practical or necessary to perform a complete biomechanical analysis. However, a comprehensive knowledge of both biomechanics and anatomy is needed to understand how the musculoskeletal system functions. Biomechanics can also be useful in a critical evaluation of current or newly proposed patient evaluations and treatments. Finally, a fundamental understanding of biomechanics is necessary to understand some of the terminology associated with kinesiology (e.g., torque, moment, moment arms).

The purposes of this chapter are to

- Review some of the basic mathematical principles used in biomechanics
- Describe forces and moments
- Discuss principles of static analysis
- Present the basic concepts in kinematics and kinetics

The analysis is restricted to the study of rigid bodies. Deformable bodies are discussed in Chapters 2 to 6. The material in this chapter is an important reference for the force analysis chapters throughout the text.

Mathematical Overview

This section is intended as a review of some of the basic mathematical concepts used in biomechanics. Although it can be skipped if the reader is familiar with this material, it would be helpful to at least review this section.

Units of Measurement

The importance of including units with measurements cannot be emphasized enough. Measurements must be accompanied by a unit for them to have any physical meaning. Sometimes, there are situations when certain units are assumed. If a clinician asks for a patient's height and the reply is "5-6," it can reasonably be assumed that the patient is 5 feet, 6 inches tall. However, that interpretation would be inaccurate if the patient was in Europe, where the metric system is used. There are also situations where the lack of a unit makes a number completely useless. If a patient was told to perform a series of exercises for two, the patient would have no idea if that meant 2 days, weeks, months, or even years.

The units used in biomechanics can be divided into two categories. First, there are the four **fundamental units** of length, mass, time, and temperature, which are defined on the basis of universally accepted standards. Every other unit is considered a **derived unit** and can be defined in terms of these fundamental units. For example, velocity is equal to length divided by time, and force is equal to mass multiplied by length divided by time squared. A list of the units needed for biomechanics is found in *Table 1.1*.

Trigonometry

Since angles are so important in the analysis of the musculoskeletal system, trigonometry is a very useful biomechanics tool. The accepted unit for measuring angles in the clinic is the degree. There are 360° in a circle. If only a portion of a circle is considered, then the angle formed is some fraction of 360°. For example, a quarter of a circle subtends an angle of 90°. Although in general, the unit degree is adopted for this text, angles also can be described in terms of radians. Since there are 2π radians in a circle, there are 57.3° per radian. When using a calculator, it is important to determine if it is set to use degrees or radians. Additionally, some computer programs, such as Microsoft Excel, use radians to perform trigonometric calculations.

Trigonometric functions are very useful in biomechanics for resolving forces into their components by relating angles to distances in a right triangle (a triangle containing a 90° angle). The most basic of these relationships (**sine, cosine,** and **tangent**) are illustrated in *Figure 1.1A*. A simple mnemonic to help remember these equations is **sohcahtoa**—sine is the opposite side divided by the hypotenuse, cosine is the adjacent side divided by the hypotenuse, and tangent is the opposite side divided by the adjacent side. Although most calculators can be used to evaluate these functions, some important values worth remembering are

$$\sin(0°) = 0, \quad \sin(90°) = 1 \qquad \text{(Equation 1.1)}$$

$$\cos(0°) = 1, \quad \cos(90°) = 0 \qquad \text{(Equation 1.2)}$$

$$\tan(45°) = 1 \qquad \text{(Equation 1.3)}$$

TABLE 1.1 **Units Used in Biomechanics**

Quantity	Metric	British	Conversion
Length	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Mass	Kilogram (kg)	Slug	1 slug = 14.59 kg
Time	Second (s)	Second (s)	1 s = 1 s
Temperature	Celsius (°C)	Fahrenheit (°F)	°F = (9/5) × °C + 32°
Force	Newton (N = kg × m/s^2)	Pound (lb = slug × ft/s^2)	1 lb = 4.448 N
Pressure	Pascal (Pa = N/m^2)	Pounds per square inch (psi = lb/in^2)	1 psi = 6895 Pa
Energy	Joule (J = N × m)	Foot pounds (ft-lb)	1 ft-lb = 1.356 J
Power	Watt (W = J/s)	Horsepower (hp)	1 hp = 7457 W

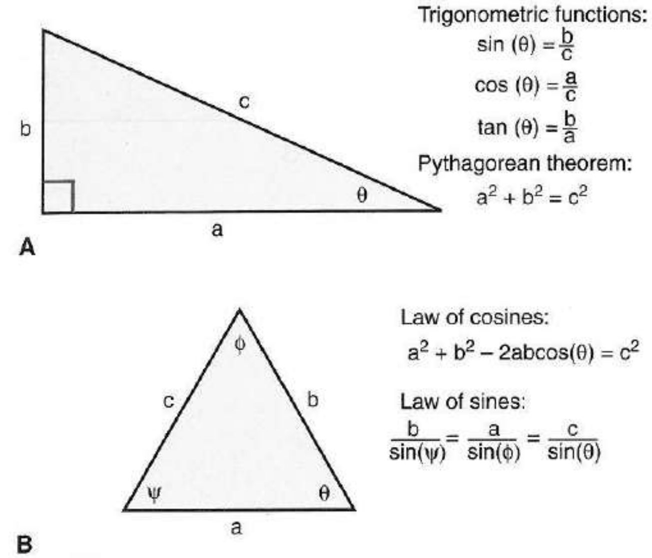


Figure 1.1 Basic trigonometric relationships. These are some of the basic trigonometric relationships that are useful for biomechanics. **A.** A right triangle. **B.** A general triangle.

Additionally, the Pythagorean theorem states that for a right triangle, the sum of the squares of the sides forming the right angle equals the square of the hypotenuse (Fig. 1.1A). Although less commonly used, there are also equations that relate angles and side lengths for triangles that do not contain a right angle (Fig. 1.1B).

Vector Analysis

Biomechanical parameters can be represented as either **scalar** or **vector** quantities. A scalar is simply represented by its magnitude. Mass, time, and length are examples of scalar quantities. A vector is generally described as having both **magnitude** and **orientation.** Additionally, a complete description of a vector also includes its **direction** (or **sense**) and **point of application**. Forces and moments are examples of vector quantities. Consider the situation of a 160-lb man sitting in a chair for 10 seconds. The force that his weight is exerting on the chair is represented by a vector with magnitude (160 lb), orientation (vertical), direction (downward), and point of application (the chair seat). However, the time spent in the chair is a scalar quantity and can be represented by its magnitude (10 seconds).

To avoid confusion, throughout this text, bolded notation is used to distinguish vectors (**A**) from scalars (B). Alternative notations for vectors found in the literature (and in classrooms, where it is difficult to bold letters) include putting a line under the letter ($\underline{A}$), a line over the letter $\bar{A}$, or an arrow over the letter $\vec{A}$. The **magnitude** of a given vector (**A**) is represented by the same letter, but not bolded (A).

By far, the most common use of vectors in biomechanics is to represent forces, such as muscle, joint reaction, and resistance forces. These vectors can be represented graphically with the use of a line with an arrow at one end (*Fig. 1.2A*). The length of the line represents its magnitude, the angular position of the line

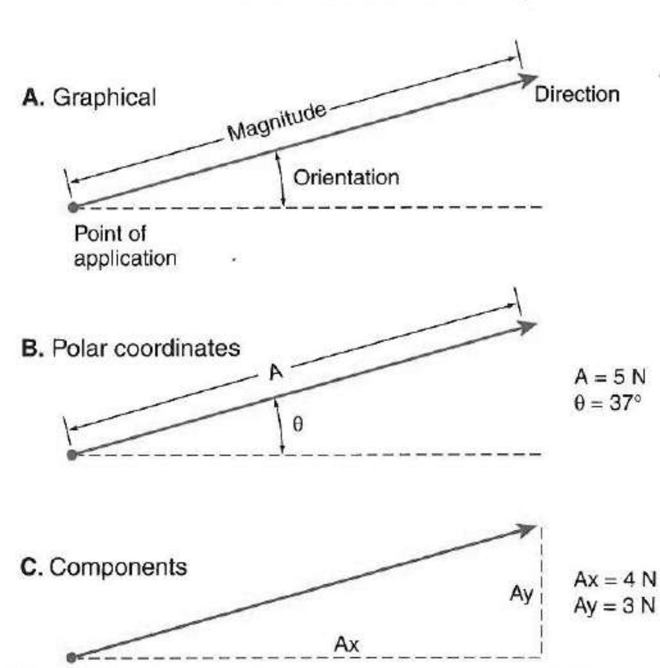


Figure 1.2 Vectors. **A.** In general, a vector has a magnitude, orientation, point of application, and direction. Sometimes the point of application is not specifically indicated in the figure. **B.** A polar coordinate representation. **C.** A component representation.

represents its orientation, the location of the arrowhead represents its direction, and the location of the line in space represents its point of application. Alternatively, this same vector can be represented mathematically with the use of either **polar coordinates** or **component resolution.** Polar coordinates represent the magnitude and orientation of the vector directly. In polar coordinates, the same vector would be 5 N at 37° from horizontal (Fig. 1.2B). With components, the vector is resolved into its relative contributions from both axes. In this example, vector **A** is resolved into its components: A_X = 4 N and A_Y = 3 N (Fig. 1.2C). It is often useful to break down vectors into components that are aligned with anatomical directions. For instance, the *x* and *y* axes may correspond to superior and anterior directions, respectively.

Although graphical representations of vectors are useful for visualization purposes, analytical representations are more convenient when adding and multiplying vectors.

Note that the directional information (up and to the right) of the vector is also embedded in this information. A vector with the same magnitude and orientation as the vector represented in Figure 1.2C, but with the opposite direction (down and to the left) is represented by A_X = –4 N and A_Y = –3 N, or 5 N at 217°. The description of the point-of-application information is discussed later in this chapter.

VECTOR ADDITION

When studying musculoskeletal biomechanics, it is common to have more than one force to consider. Therefore, it is important to understand how to work with more than one vector. When adding or subtracting two vectors, there are some important properties to consider. Vector addition is commutative:

$$A + B = B + A \qquad \text{(Equation 1.4)}$$

$$A - B = A + (-B) \qquad \text{(Equation 1.5)}$$

Vector addition is associative:

$$A + (B + C) = (A + B) + C \qquad \text{(Equation 1.6)}$$

Unlike scalars, which can just be added together, both the magnitude and orientation of a vector must be taken into account. The detailed procedure for adding two vectors (**A** + **B** = **C**) is shown in *Examining the Forces Box 1.1* for the graphical, polar coordinate, and component representation of vectors. The graphical representation uses the "tip to tail" method. The first step is to draw the first vector, **A**. Then the second vector, **B**, is drawn so that its tail sits on the tip of the first vector. The vector representing the sum of these two vectors (**C**) is obtained by connecting the tail of vector **A** and the tip of vector **B**. Since vector addition is commutative, the same solution would have been obtained if vector **B** were the first vector. When using polar coordinates, the vectors are drawn as in the graphical method and then the law of cosines is used to determine the magnitude of **C** and the law of sines is used to determine the direction of **C** (see Fig. 1.1 for definitions of these laws).

For the component resolution method, each vector is broken down into its respective x and y components. The components represent the magnitude of the vector in that direction. The x and y components are summed:

$$C_X = A_X + B_X \qquad \text{(Equation 1.7)}$$

$$C_Y = A_Y + B_Y \qquad \text{(Equation 1.8)}$$

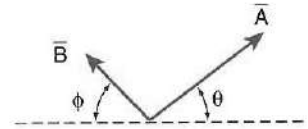
The vector **C** can either be left in terms of its components, C_X and C_Y, or be converted into a magnitude, C, using the Pythagorean theorem, and orientation, θ, using trigonometry. This method is the most efficient of the three presented and is used throughout the text.

VECTOR MULTIPLICATION

Multiplication of a vector by a scalar is relatively straightforward. Essentially, each component of the vector is individually multiplied by the scalar, resulting in another vector. For example, if the vector in Figure 1.2 is multiplied by 5, the result is $A_X = 5 \times 4$ N = 20 N and $A_Y = 5 \times 3$ N = 15 N. Another form of vector multiplication is the **cross product**, in which two vectors are multiplied together, resulting in another vector (**C** = **A** × **B**). The orientation of **C** is such that it is mutually perpendicular to **A** and **B**. The right hand rule is used to determine the direction of **C**. As the fingers sweep from **A** to **B**, the thumb is pointed in the direction of the vector **C** (up in this case). The magnitude of **C** is calculated as C = A × B sin (θ), where θ represents the angle between A and B, and × denotes scalar multiplication. These relationships are illustrated in *Figure 1.3*. The cross product is used for calculating joint torques later in this chapter.

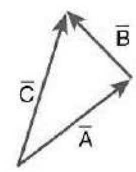
EXAMINING THE FORCES BOX 1.1
Addition of Two Vectors

Addition of 2 vectors: $\bar{A} + \bar{B}$

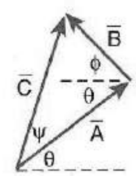


A = 5N
B = 2.8N
θ = 37°
ϕ = 45°

Case A: Graphical



Case B: Polar

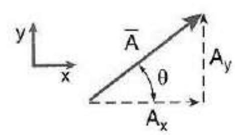


Law of cosines:
$A^2 + B^2 - 2AB\cos(\theta + \phi) = C^2$
C = 5.4N

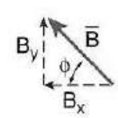
Law of sines:
$\frac{B}{\sin \Psi} = \frac{C}{\sin(\theta + \phi)}$
ψ = 31°

Therefore, the angle that C makes with a horizontal axis is 68° (= ψ + θ)

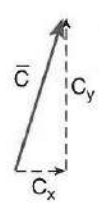
Case C: Components



$A_x = A\cos(\theta) = 4N$
$A_y = A\sin(\theta) = 3N$



$B_x = -B\cos(\phi) = -2N$
$B_y = B\sin(\phi) = 2N$



$C_x = A_x + B_x = 2N$
$C_y = A_y + B_y = 5N$

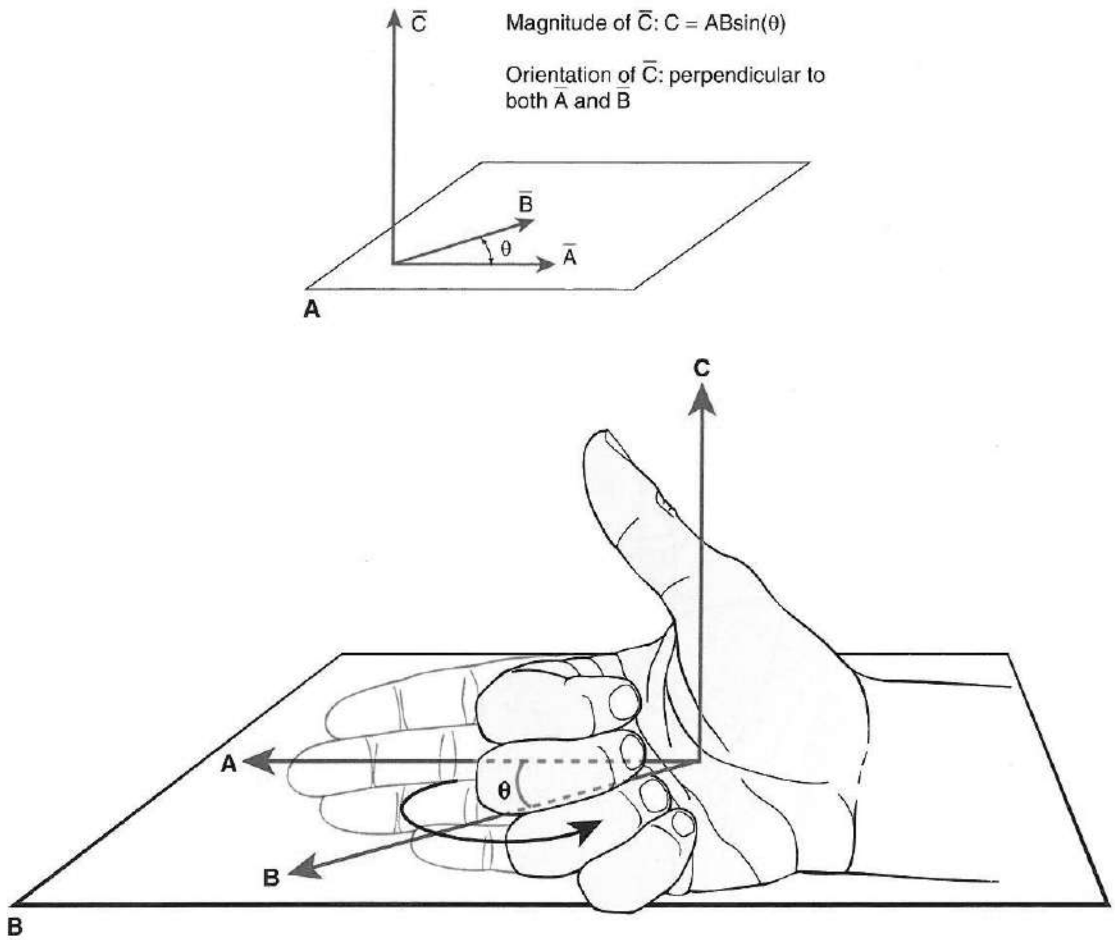


Figure 1.3 Vector cross product. **C** is shown as the cross product of **A** and **B**. Note that **A** and **B** could be any two vectors in the indicated plane and C would still have the same orientation.

Coordinate Systems

A three-dimensional analysis is necessary for a complete representation of human motion. Such analyses require a coordinate system, which is typically composed of anatomically aligned axes: medial/lateral (ML), anterior/posterior (AP), and superior/inferior (SI). It is often convenient to consider only a two-dimensional, or planar, analysis, in which only two of the three axes are considered. In the human body, there are three perpendicular anatomical planes, which are referred to as the **cardinal planes**. The **sagittal plane** is formed by the SI and AP axes, the **frontal (or coronal) plane** is formed by the SI and ML axes, and the **transverse plane** is formed by the AP and ML axes (*Fig. 1.4*).

The motion of any bone can be referenced with respect to either a **local** or a **global** coordinate system. For example, the motion of the tibia can be described by how it moves with respect to the femur (local coordinate system) or how it moves with respect to the room (global coordinate system). Local coordinate systems are useful for understanding joint function and assessing range of motion, while global coordinate systems are useful when functional activities are considered.

Most of this text focuses on two-dimensional analyses, for several reasons. First, it is difficult to display three-dimensional information on the two-dimensional pages of a book. Additionally, the mathematical analysis for a three-dimensional problem is very complex. Perhaps the most important reason is that the fundamental biomechanical principles in a two-dimensional analysis are the same as those in a three-dimensional analysis. It is therefore possible to use a simplified two-dimensional representation of a three-dimensional problem to help explain a concept with minimal mathematical complexity (or at least less complexity).

Forces and Moments

The musculoskeletal system is responsible for generating forces that move the human body in space as well as preventing unwanted motion. Understanding the mechanics and pathomechanics of human motion requires an ability to study the forces and moments applied to, and generated by, the body or a particular body segment.

Forces

The reader may have a conceptual idea about what a force is but find it difficult to come up with a formal definition. For the purposes of this text, a **force** is defined as a "push or pull" that results from physical contact between two objects. The only exception to this rule that is considered in this text is the force due to gravity, in which there is no direct physical contact between two objects. Some of the

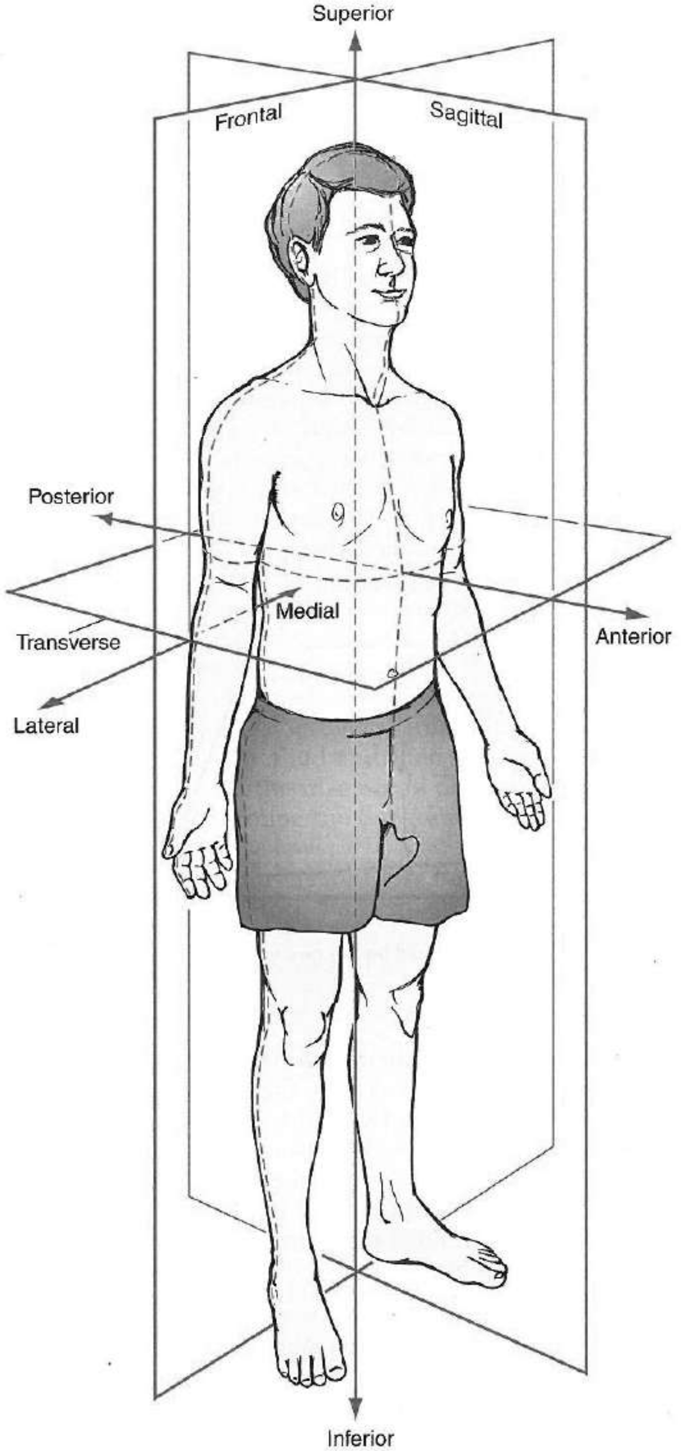


Figure 1.4 Cardinal planes. The cardinal planes, sagittal, frontal, and transverse, are useful reference frames in a three-dimensional representation of the body. In two-dimensional analyses, the sagittal plane is the common reference frame.

more common force generators with respect to the musculoskeletal system include muscles, tendons, ligaments, friction, ground reaction, and weight.

A distinction must be made between the **mass** and the **weight** of a body. The mass of an object is defined as the amount of matter composing that object. The weight of an object is the force acting on that object due to gravity and is the product of its mass and the acceleration due to gravity ($g = 9.8\ m/s^2$). So, while an object's mass is the same on Earth as it is on the moon, its weight on the moon is less, since the acceleration due to gravity is lower on the moon. This distinction is important in biomechanics, not to help plan a trip to the moon, but for ensuring that a unit of mass is not treated as a unit of force.

As mentioned previously, force is a vector quantity with magnitude, orientation, direction, and a point of application. *Figure 1.5* depicts several forces acting on the leg in the frontal plane during stance. The forces from the abductor and adductor muscles act through their tendinous insertions, while the hip joint reaction force acts

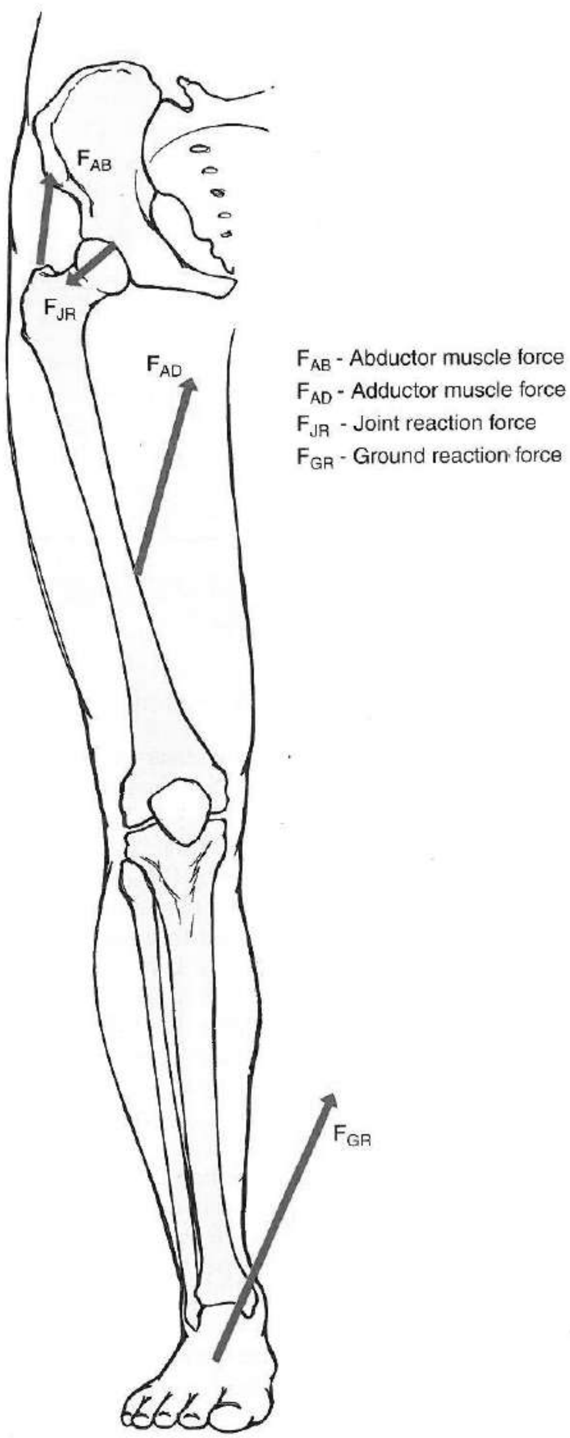


Figure 1.5 Vectors in anatomy. Example of how vectors can be combined with anatomical detail to represent the action of forces. Some of the forces acting on the leg are shown here.

through its respective joint center of rotation. In general, the point of application of a force (e.g., tendon insertion) is located with respect to a fixed point on a body, usually the joint center of rotation. This information is used to calculate the **moment** due to that force.

Moments

In kinesiology, a moment (M) is typically caused by a force (F) acting at a distance (r) from the center of rotation of a segment. A moment tends to cause a rotation and is defined by the cross product function: **M = r × F.** Therefore, a moment is represented by a vector that passes through the point of interest (e.g., the center of rotation) and is perpendicular to both the force and distance vectors (*Fig. 1.6*). For a two-dimensional analysis, both the force and distance vectors are in the plane of the paper, so the moment vector is always directed perpendicular to the page, with a line of action through the point of interest. Since it has only this one orientation and line of action, a moment is often treated as a scalar quantity in a two-dimensional analysis, with only magnitude and direction. **Torque** is another term that is synonymous with a scalar moment. From the definition of a cross product, the magnitude of a moment (or torque) is calculated as $M = r \times F \times \sin(\theta)$. Its direction is referred to as the direction in which it would tend to cause an object to rotate (*Fig. 1.7A*).

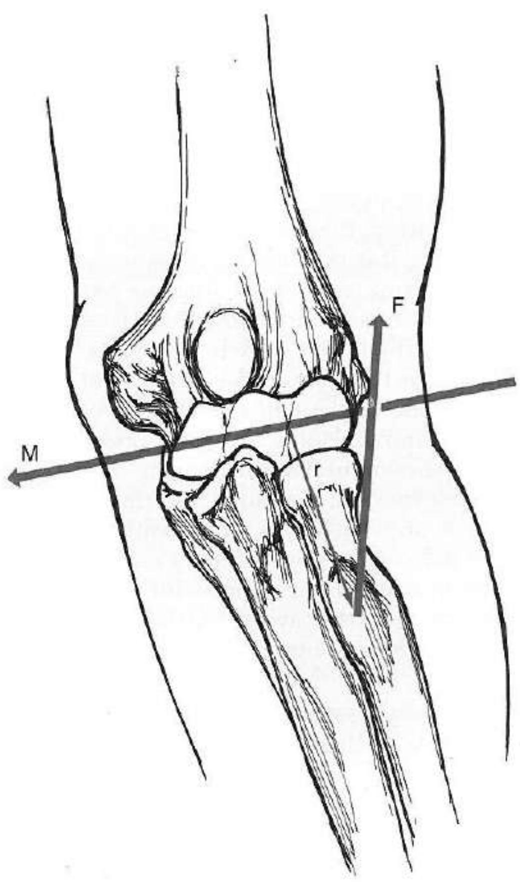
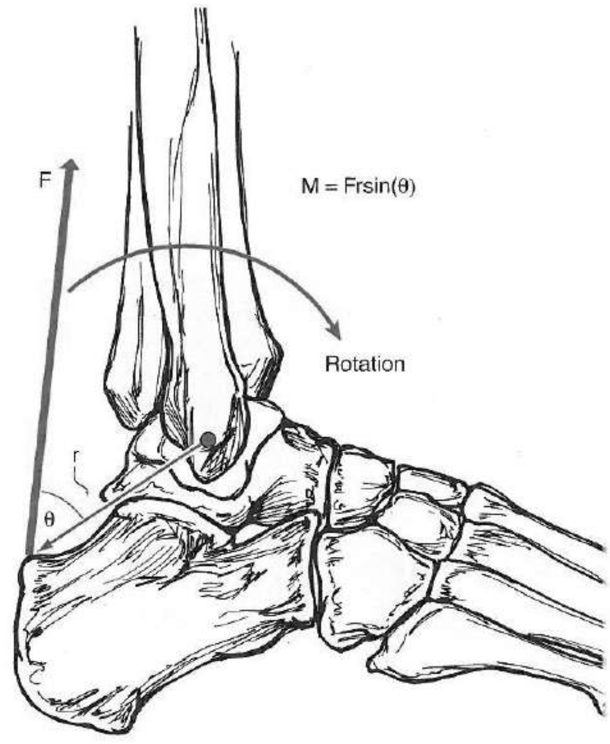
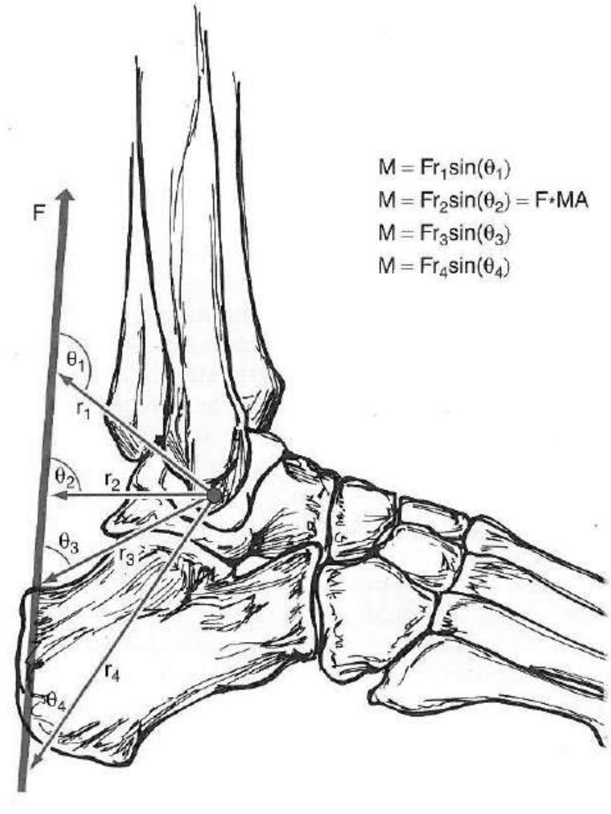


Figure 1.6 Three-dimensional moment analysis. The moment acting on the elbow from the force of the biceps is shown as a vector aligned with the axis of rotation. *F*, force vector; r, distance from force vector to joint COR; *M*, moment vector.



A



B

Figure 1.7 Two-dimensional moment analysis. **A.** Plantar flexion moment created by force at the Achilles tendon. **B.** Note that no matter which distance vector is chosen, the value for the moment is the same.

(*Figure is continued on next page*)

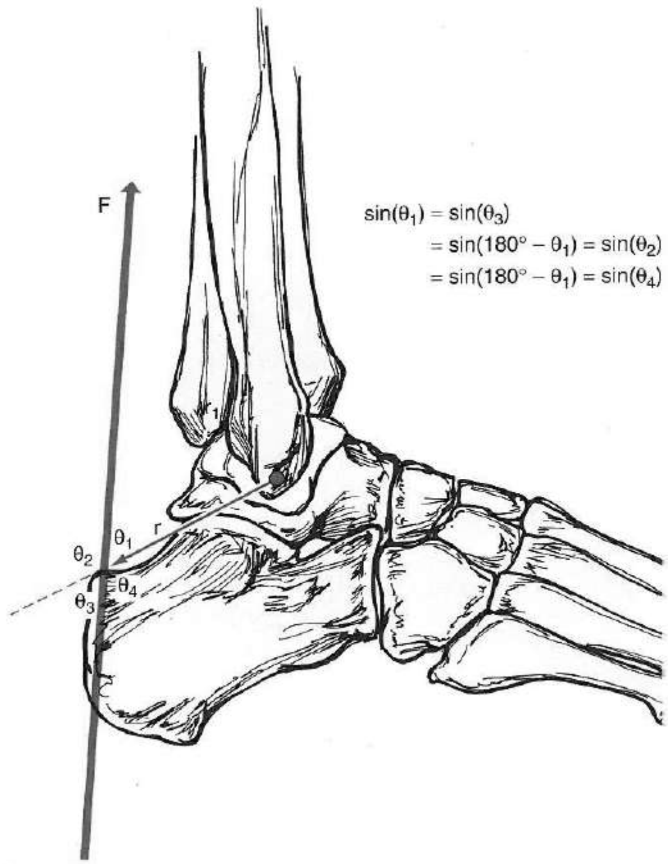


Figure 1.7 (*Continued*) **C.** Also, no matter which angle is chosen, the value for the sine of the angle is the same, so the moment is the same.

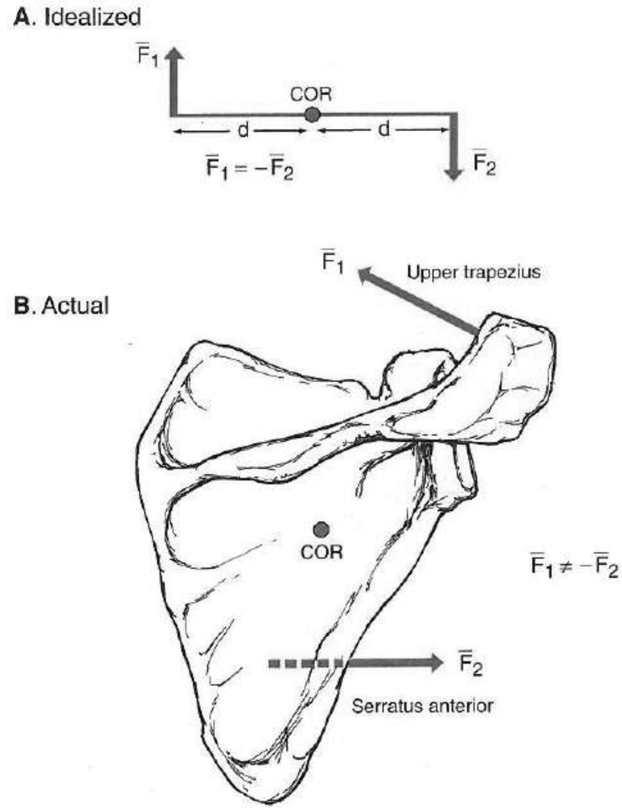


Figure 1.8 Force couples. Distinction between an idealized force couple **(A)** and a more realistic one **(B)**. Even though the scapular example given is not a true force couple, it is typically referred to as one. *COR*, center of rotation.

Although there are several different distances that can be used to connect a vector and a point, **the same moment is calculated no matter which distance is selected** (Fig. 1.7B). The distance that is perpendicular to the force vector is referred to as the **moment arm** (MA) of that force (r_2 in Fig. 1.7B). Since the sine of 90° is equal to 1, the use of a moment arm simplifies the calculation of moment to $M = MA \times F$. The moment arm can also be calculated from any distance as $MA = r \times \sin(\theta)$. Additionally, although there are four separate angles between the force and distance vectors, all four angles result in the same moment calculation (Fig. 1.7C).

The examples in Figures 1.6 and 1.7 have both force and moment components. However, consider the situation in *Figure 1.8A*. Although the two applied forces create a moment, they have the same magnitude and orientation but opposite directions. Therefore, their vector sum is zero. This is an example of a **force couple.** A pure force couple results in rotational motion only, since there are no unbalanced forces. In the musculoskeletal system, all of these conditions are seldom met, so pure force couples are rare. In general, muscles are responsible for producing both forces and moments, thus resulting in both translational and rotational motion. However, there are examples in the human body in which two or more muscles work in concert to produce a moment, such as the upper trapezius and serratus anterior (Fig. 1.8B). Although these muscles do not have identical magnitudes or orientations, this situation is frequently referred to as a force couple.

Muscle Forces

As mentioned previously, there are three important parameters to consider with respect to the force of a muscle: orientation, magnitude, and point of application. With some care, it is possible to measure orientation and line of action from cadavers or imaging techniques such as magnetic resonance imaging (MRI) and computed tomography (CT) [1,3]. This information is helpful in determining the function and efficiency of a muscle in producing a moment. As an example, two muscles that span the glenohumeral joint, the supraspinatus and middle deltoid, are shown in Examining the Forces Box 1.2. From the information provided for muscle orientation and point of application in this position, the moment arm of the deltoid is approximately equal to that of the supraspinatus, even though the deltoid insertion on the humerus is much farther away from the center of rotation than the supraspinatus insertion.

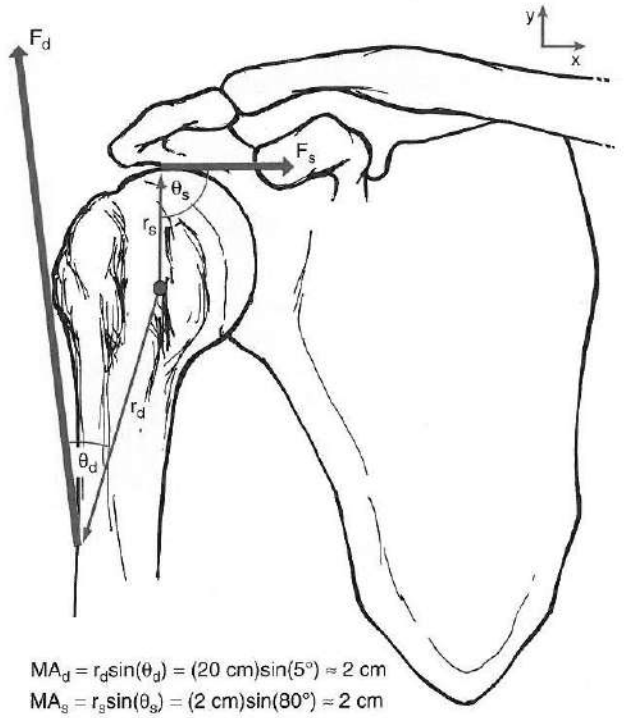
CLINICAL RELEVANCE

MUSCLE FORCES

In addition to generating moments that are responsible for **angular motion** (rotation), muscles also produce forces that can cause **linear motion** (translation). This force can be either a stabilizing or a destabilizing force. For example,

(*continued*)

EXAMINING THE FORCES BOX 1.2
Moment Arms of the Deltoid (MA_d) and the Supraspinatus (MA_s)



(Continued)

since the supraspinatus orientation shown in *Examining the Forces Box 1.2* is primarily directed medially, it tends to pull the humeral head into the glenoid fossa. This compressive force helps stabilize the glenohumeral joint. However, since the deltoid orientation is directed superiorly, it tends to produce a destabilizing force that may result in superior translation of the humeral head.

These analyses are useful, since they can be performed even if the magnitude of a muscle's force is unknown. However, to understand a muscle's function completely, its force magnitude must be known. Although forces can be measured with invasive force transducers [13], instrumented arthroplasty systems [6], or simulations in cadaver models [9], there are currently no noninvasive experimental methods that can be used to measure the in vivo force of intact muscles. Consequently, basic concepts borrowed from freshman physics can be used to predict muscle forces. Although they often involve many simplifying assumptions, such methods can be very useful in understanding joint mechanics and are presented in the next section.

Statics

Statics is the study of the forces acting on a body at rest or moving with a constant velocity. Although the human body is almost always accelerating, a static analysis offers a simple method of addressing musculoskeletal problems. This analysis may either solve the problem or provide a basis for a more sophisticated dynamic analysis.

Newton's Laws

Since the musculoskeletal system is simply a series of objects in contact with each other, some of the basic physics principles developed by Sir Isaac Newton (1642–1727) are useful. Newton's laws are as follows:

First law: An object remains at rest (or continues moving at a constant velocity) unless acted upon by an unbalanced external force.
Second law: If there is an unbalanced force acting on an object, it produces an acceleration in the direction of the force, directly proportional to the force ($f = ma$).
Third law: For every action (force) there is a reaction (opposing force) of equal magnitude but in the opposite direction.

From Newton's first law, it is clear that if a body is at rest, there can be no unbalanced external forces acting on it. In this situation, termed **static equilibrium**, all of the external forces acting on a body must add (in a vector sense) to zero. An extension of this law to objects larger than a particle is that the sum of the external moments acting on that body must also be equal to zero for the body to be at rest. Therefore, for a three-dimensional analysis, there are a total of six equations that must be satisfied for static equilibrium:

$$\begin{matrix} \Sigma F_X = 0 & \Sigma F_Y = 0 & \Sigma F_Z = 0 \\ \Sigma M_X = 0 & \Sigma M_Y = 0 & \Sigma M_Z = 0 \end{matrix} \qquad \text{(Equation 1.9)}$$

For a two-dimensional analysis (in the *x–y* plane), there are only two in-plane force components and one perpendicular moment (torque) component:

$$\Sigma F_X = 0 \quad \Sigma F_Y = 0 \quad \Sigma M_Z = 0 \qquad \text{(Equation 1.10)}$$

Under many conditions, it is reasonable to assume that all body parts are in a state of static equilibrium, and these three equations can be used to calculate some of the forces acting on the musculoskeletal system. When a body is not in static equilibrium, Newton's second law states that any unbalanced forces and moments are proportional to the acceleration of the body. That situation is considered later in this chapter.

Solving Problems

A general approach used to solve for forces during static equilibrium is as follows:

Step 1 Isolate the body of interest.
Step 2 Sketch this body and all external forces (referred to as a **free-body diagram**).
Step 3 Sum the forces and moments equal to zero.
Step 4 Solve for the unknown forces.